Final Project

problem Formulation and Set up.

~includes belief update and all formulations needed to solve problem for 2 systems A + B

System A : Accelerometer always on

$S: \{1, 2, 3, 4\}$

(Sit, Stand, walk, run)

$U: \{1, 2, 3\}$

1: turn off filter

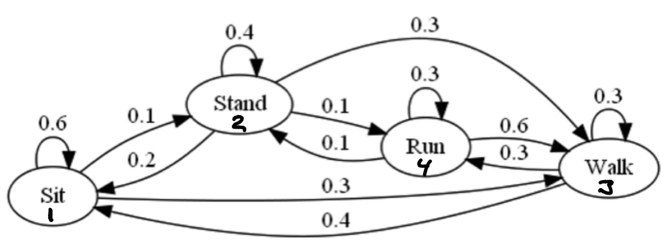
2: turn on filter 1

3: turn on filter 2

$Y: \{1, 2, 3, 4\}$

~ From accelerometer data

Transition Probabilities



From thatte et al

Fig. 1. Markov chain of four physical activities {*Sit*, *Stand*, *Run*, *Walk*} [5].

$\mathbb{P}(S_{k+1} = j \mid S_k = i, U_k = u)$

independant of action

$\mathbb{P}(S_{k+1} = j \mid S_k = i)$

$$\begin{array}{cc} & j \rightarrow \\ i \downarrow & \begin{bmatrix} .6 & .1 & .3 & 0 \\ .2 & .4 & .3 & .1 \\ .4 & 0 & .3 & .3 \\ 0 & .1 & .6 & .3 \end{bmatrix} \end{array}$$

Reward Model

- using a filter costs energy ($-e_{filter}$)
- picking the correct action gets a reward (+)
- Accelerometer always on ($-e_{accel}$)

$r(s,u)$

$$r(s,1) = \begin{cases} 0 - e_{accel} & s = 3,4 \\ R - e_{accel} & s = 1,2 \end{cases} \quad \text{(No Filter)}$$

$$r(s,2) = \begin{cases} R - e_{filt} - e_{accel} & s = 3 \\ -e_{filt} - e_{accel} & s = 1,2,4 \end{cases} \quad \text{(Filter 1)}$$

$$r(s,3) = \begin{cases} R - e_{filt} - e_{accel} & s = 4 \\ -e_{filt} - e_{accel} & s = 1,2,3 \end{cases} \quad \text{(Filter 2)}$$

$$\begin{cases} u = 1 & i = 1,2 \quad i \neq 1,2 \\ u = 2,3 & i = 4 \\ & i \neq 4 \end{cases}$$

Observation Model

$$\mathbb{P}(s_{k+1}=j, Y_k=y \mid s_k=i, U_k=u) = \mathbb{P}(Y_k=y \mid s_{k+1}=j, s_k=i, U_k=u)\,\mathbb{P}(s_{k+1}=j \mid s_k=i)$$

$\Rightarrow$ independant of action and next state

$$\mathbb{P}(s_{k+1}=j, Y_k=y \mid s_k=i) = \mathbb{P}(Y_k=y \mid s_k=i)\,\mathbb{P}(s_{k+1}=j \mid s_k=i)$$

Define $\mathbb{P}(Y_k=y \mid s_k=i) = \begin{cases} \varepsilon/3 & y \neq i \\ \varepsilon/3 & y \neq i \\ \varepsilon/3 & y \neq i \\ 1-\varepsilon & y = i \end{cases}$

ex: $i=2$, $y=1$

$\varepsilon/3$ chance for $y=2,3,4$

Belief update

$$\beta_{k+1} = \mathbb{B}(\beta_k, U_k, Y_k)$$

$$\beta_{k+1} = \begin{bmatrix} \beta_{k+1}(1) \\ \beta_{k+1}(2) \\ \beta_{k+1}(3) \\ \beta_{k+1}(4) \end{bmatrix} \qquad \beta_{k+1}(j) = \mathbb{P}(s_{k+1}=j \mid s_k \sim \beta_k, U_k=u, Y_k=y)$$

Belief update eqn

$$\beta_{k+1}(j) = \frac{\sum_i \beta_k(i)\,\mathbb{P}(Y_k=y \mid \cancel{s_{k+1}=j}, s_k=i, \cancel{U_k=u})\,\mathbb{P}(s_{k+1}=j \mid s_k=i, \cancel{U_k=u})}{\sum_i \beta_k(i)\,\mathbb{P}(Y_k=y \mid s_k=i, \cancel{U_k=u})}$$

Initialize β for $1 \cdots Q$

$$\sum_{i=1}^{4} \beta(i) = 1$$

- Could do random values
  ~ requires larger Q value due to the randomness

$$\left[\frac{(l-1)}{Q-1} ; \frac{(Q-l)}{Q-1} \right]$$

$1 \rightarrow 0$

$0 \rightarrow 1$

$$\frac{i_1}{Q} \quad \frac{i_2}{Q} \quad \frac{i_3}{Q}$$

sometimes set

[same # $\rightarrow$]

for A_0 in each row

(diff. per row, same per column)

- Relation between Q & $\vec{B}$.

  Why use higher Q?

  random save results ≃ seen in HW2
  w/ Q = 1000
  (not big change w/ other Q vals)

$B = \text{rand}(2, Q)$

$$= \frac{\sum_{i=0}^{4} \beta_k(i) \overbrace{\mathbb{P}(Y_k = y \mid s_k = i)}^{P_o} \overbrace{\mathbb{P}(s_{k+1} = j \mid s_k = i)}^{P_t \to \text{transition prob.}}}{\sum_{i=0}^{4} \beta_k(i) \underbrace{\mathbb{P}(Y_k = y \mid s_k = i)}_{P_o \;=\; \text{Observat. model}}}$$

General Belief Function

$$\beta_{k+1}(j) = \frac{\beta_k(1)P_o(1)P_t(1) + \beta_k(2)P_o(2)P_t(2) + \beta_k(3)P_o(3)P_t(3) + \beta_k(4)P_o(4)P_t(4)}{\beta_k(1)P_o(1) + \beta_k(2)P_o(2) + \beta_k(3)P_o(3) + \beta_k(4)P_o(4)}$$

where $j = 1 \dots 4$

$P_o(i) = \mathbb{P}(Y_k = y \mid s_k = i)$

$P_t(i) = \mathbb{P}(s_{k+1} = j \mid s_k = i)$

& $A_k = [\alpha^1 \dots \alpha^Q]$

$\alpha = \begin{bmatrix} - \\ - \\ - \end{bmatrix}_{4 \times 1}$

so $\beta_{k+1}(j)$ 4 cases $1 \dots 4$

per each case, $\beta_{k+1}(j)$

has 4 more cases $y = 1 \dots 4$

ie $\beta_{k+1}(1) = \dots P_t(1 \mid i)$

and $y =$ 1

$(P_o(y \mid i))$ 2

3

4

16 possible

$$\beta_{k+1}^{i}(j) = \frac{\beta_k(1)P_o(1)P_t(1)+\beta_k(2)P_o(2)P_t(2)+\beta_k(3)P_o(3)P_t(3)+\beta_k(4)P_o(4)P_t(4)}{\beta_k(1)P_o(1)+\beta_k(2)P_o(2)+\beta_k(3)P_o(3)+\beta_k(4)P_o(4)}$$

j=1 (sit)

y=1

$$\beta_{k+1}(1) = \frac{\beta_k(1)\overset{\text{sit}\to\text{sit}}{\underset{y=i}{(1-\varepsilon)}}(.6)+\beta_k(2)\overset{\text{stand}\to\text{sit}}{\underset{y\neq i}{\left(\frac{\varepsilon}{3}\right)}}(.2)+\beta_k(3)\overset{\text{walk}\to\text{sit}}{\underset{y\neq i}{\left(\frac{\varepsilon}{3}\right)}}(.4)+\overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)\left(\frac{\varepsilon}{3}\right)}}(0)}{\beta_k(1)(1-\varepsilon)+\beta_k(2)\left(\frac{\varepsilon}{3}\right)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

y=2

$$\beta_{k+1}(1) = \frac{\beta_k(1)\overset{\text{sit}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.6)+\beta_k(2)\overset{\text{stand}\to\text{sit}}{(1-\varepsilon)}(.2)+\beta_k(3)\overset{\text{walk}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.4)+\overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)\left(\frac{\varepsilon}{3}\right)}}(0)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)(1-\varepsilon)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

y=3

$$\beta_{k+1}(1) = \frac{\beta_k(1)\overset{\text{sit}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.6)+\beta_k(2)\overset{\text{stand}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.2)+\beta_k(3)\overset{\text{walk}\to\text{sit}}{(1-\varepsilon)}(.4)+\overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)\left(\frac{\varepsilon}{3}\right)}}(0)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)\left(\frac{\varepsilon}{3}\right)+\beta_k(3)(1-\varepsilon)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

y=4

$$\beta_{k+1}(1) = \frac{\beta_k(1)\overset{\text{sit}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.6)+\beta_k(2)\overset{\text{stand}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.2)+\beta_k(3)\overset{\text{walk}\to\text{sit}}{\left(\frac{\varepsilon}{3}\right)}(.4)+\overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)(1-\varepsilon)}}(0)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)\left(\frac{\varepsilon}{3}\right)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)(1-\varepsilon)}$$

j=2 (stand)

y=1

$$\beta_{k+1}(2) = \frac{\overset{\text{sit}\to\text{stand}}{\underset{y=i}{\beta_k(1)(1-\varepsilon)(.1)}} + \overset{\text{stand}\to\text{stand}}{\underset{y\neq i}{\beta_k(2)(\tfrac{\varepsilon}{3})(.4)}} + \overset{\text{walk}\to\text{stand}}{\underset{y\neq i}{\beta_k(3)(\tfrac{\varepsilon}{3})(0)}} + \overset{\text{run}\to\text{stand}}{\beta_k(4)(\tfrac{\varepsilon}{3})(.1)}}{\beta_k(1)(1-\varepsilon) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

y=2

$$\beta_{k+1}(2) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.1) + \beta_k(2)(1-\varepsilon)(.4) + \beta_k(3)(\tfrac{\varepsilon}{3})(0) + \beta_k(4)(\tfrac{\varepsilon}{3})(.1)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(1-\varepsilon) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

y=3

$$\beta_{k+1}(2) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.1) + \beta_k(2)(\tfrac{\varepsilon}{3})(.4) + \beta_k(3)(1-\varepsilon)(0) + \beta_k(4)(\tfrac{\varepsilon}{3})(.1)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(1-\varepsilon) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

y=4

$$\beta_{k+1}(2) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.1) + \beta_k(2)(\tfrac{\varepsilon}{3})(.4) + \beta_k(3)(\tfrac{\varepsilon}{3})(0) + \beta_k(4)(1-\varepsilon)(.1)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(1-\varepsilon)}$$

$j=3$ (walk)

$y=1$

$$\beta_{k+1}(3)=\frac{\overset{\text{sit}\to\text{walk}}{\underset{y=i}{\beta_k(1)(1-\varepsilon)(.3)}}+\overset{\text{stand}\to\text{walk}}{\underset{y\neq i}{\beta_k(2)(\tfrac{\varepsilon}{3})(.3)}}+\overset{\text{walk}\to\text{walk}}{\underset{y\neq i}{\beta_k(3)(\tfrac{\varepsilon}{3})(.3)}}+\overset{\text{run}\to\text{walk}}{\beta_k(4)(\tfrac{\varepsilon}{3})(.6)}}{\beta_k(1)(1-\varepsilon)+\beta_k(2)(\tfrac{\varepsilon}{3})+\beta_k(3)(\tfrac{\varepsilon}{3})+\beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=2$

$$\beta_{k+1}(3)=\frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.3)+\beta_k(2)(1-\varepsilon)(.3)+\beta_k(3)(\tfrac{\varepsilon}{3})(.3)+\beta_k(4)(\tfrac{\varepsilon}{3})(.6)}{\beta_k(1)(\tfrac{\varepsilon}{3})+\beta_k(2)(1-\varepsilon)+\beta_k(3)(\tfrac{\varepsilon}{3})+\beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=3$

$$\beta_{k+1}(3)=\frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.3)+\beta_k(2)(\tfrac{\varepsilon}{3})(.3)+\beta_k(3)(1-\varepsilon)(.3)+\beta_k(4)(\tfrac{\varepsilon}{3})(.6)}{\beta_k(1)(\tfrac{\varepsilon}{3})+\beta_k(2)(\tfrac{\varepsilon}{3})+\beta_k(3)(1-\varepsilon)+\beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=4$

$$\beta_{k+1}(3)=\frac{\beta_k(1)(\tfrac{\varepsilon}{3})(.3)+\beta_k(2)(\tfrac{\varepsilon}{3})(.3)+\beta_k(3)(\tfrac{\varepsilon}{3})(.3)+\beta_k(4)(1-\varepsilon)(.6)}{\beta_k(1)(\tfrac{\varepsilon}{3})+\beta_k(2)(\tfrac{\varepsilon}{3})+\beta_k(3)(\tfrac{\varepsilon}{3})+\beta_k(4)(1-\varepsilon)}$$

$j=4$ (Run)

$y=1$

$$\beta_{k+1}(4) = \frac{\beta_k(1)\underbrace{\overset{\text{sit}\to\text{run}}{(1-\varepsilon)}}_{y=i}(0) + \beta_k(2)\underbrace{\overset{\text{stand}\to\text{run}}{(\tfrac{\varepsilon}{3})}}_{y\neq i}(.1) + \beta_k(3)\underbrace{\overset{\text{walk}\to\text{run}}{(\tfrac{\varepsilon}{3})}}_{y\neq i}(.3) + \beta_k(4)\overset{\text{run}\to\text{run}}{(\tfrac{\varepsilon}{3})}(.3)}{\beta_k(1)(1-\varepsilon) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=2$

$$\beta_{k+1}(4) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(0) + \beta_k(2)(1-\varepsilon)(.1) + \beta_k(3)(\tfrac{\varepsilon}{3})(.3) + \beta_k(4)(\tfrac{\varepsilon}{3})(.3)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(1-\varepsilon) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=3$

$$\beta_{k+1}(4) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(0) + \beta_k(2)(\tfrac{\varepsilon}{3})(.1) + \beta_k(3)(1-\varepsilon)(.3) + \beta_k(4)(\tfrac{\varepsilon}{3})(.3)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(1-\varepsilon) + \beta_k(4)(\tfrac{\varepsilon}{3})}$$

$y=4$

$$\beta_{k+1}(4) = \frac{\beta_k(1)(\tfrac{\varepsilon}{3})(0) + \beta_k(2)(\tfrac{\varepsilon}{3})(.1) + \beta_k(3)(\tfrac{\varepsilon}{3})(.3) + \beta_k(4)(1-\varepsilon)(.3)}{\beta_k(1)(\tfrac{\varepsilon}{3}) + \beta_k(2)(\tfrac{\varepsilon}{3}) + \beta_k(3)(\tfrac{\varepsilon}{3}) + \beta_k(4)(1-\varepsilon)}$$

$\boxed{Y = 1}$

$y=1$

$$\beta_{k+1}(1) = \frac{\overset{\text{sit}\to\text{sit}}{\beta_k(1)\underbrace{(1-\varepsilon)}_{y=i}(.6)} + \overset{\text{stand}\to\text{sit}}{\beta_k(2)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.2)} + \overset{\text{walk}\to\text{sit}}{\beta_k(3)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.4)} + \overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)(0)}}}{\beta_k(1)(1-\varepsilon)+\beta_k(2)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(3)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)}$$

$y=1$

$$\beta_{k+1}(2) = \frac{\overset{\text{sit}\to\text{stand}}{\beta_k(1)\underbrace{(1-\varepsilon)}_{y=i}(.1)} + \overset{\text{stand}\to\text{stand}}{\beta_k(2)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.4)} + \overset{\text{walk}\to\text{stand}}{\beta_k(3)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(0)} + \overset{\text{run}\to\text{stand}}{\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)(.1)}}{\beta_k(1)(1-\varepsilon)+\beta_k(2)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(3)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)}$$

$y=1$

$$\beta_{k+1}(3) = \frac{\overset{\text{sit}\to\text{walk}}{\beta_k(1)\underbrace{(1-\varepsilon)}_{y=i}(.3)} + \overset{\text{stand}\to\text{walk}}{\beta_k(2)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.3)} + \overset{\text{walk}\to\text{walk}}{\beta_k(3)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.3)} + \overset{\text{run}\to\text{walk}}{\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)(.6)}}{\beta_k(1)(1-\varepsilon)+\beta_k(2)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(3)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)}$$

$y=1$

$$\beta_{k+1}(4) = \frac{\overset{\text{sit}\to\text{run}}{\beta_k(1)\underbrace{(1-\varepsilon)}_{y=i}(0)} + \overset{\text{stand}\to\text{run}}{\beta_k(2)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.1)} + \overset{\text{walk}\to\text{run}}{\beta_k(3)\underbrace{\left(\tfrac{\varepsilon}{3}\right)}_{y\neq i}(.3)} + \overset{\text{run}\to\text{run}}{\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)(.3)}}{\beta_k(1)(1-\varepsilon)+\beta_k(2)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(3)\left(\tfrac{\varepsilon}{3}\right)+\beta_k(4)\left(\tfrac{\varepsilon}{3}\right)}$$

$\boxed{Y=2}$

$y=2$

$$\beta_{k+1}(1)=\frac{\overset{\text{sit}\to\text{sit}}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.6)}+\overset{\text{stand}\to\text{sit}}{\beta_k(2)(1-\varepsilon)(.2)}+\overset{\text{walk}\to\text{sit}}{\beta_k(3)\left(\frac{\varepsilon}{3}\right)(.4)}+\overset{\text{run}\to\text{sit}}{\cancel{\beta_k(4)\left(\frac{\varepsilon}{3}\right)}(0)}}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)(1-\varepsilon)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=2$

$$\beta_{k+1}(2)=\frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.1)+\beta_k(2)(1-\varepsilon)(.4)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)(0)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)(.1)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)(1-\varepsilon)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=2$

$$\beta_{k+1}(3)=\frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.3)+\beta_k(2)(1-\varepsilon)(.3)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)(.3)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)(.6)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)(1-\varepsilon)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=2$

$$\beta_{k+1}(4)=\frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(0)+\beta_k(2)(1-\varepsilon)(.1)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)(.3)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)(.3)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)+\beta_k(2)(1-\varepsilon)+\beta_k(3)\left(\frac{\varepsilon}{3}\right)+\beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$$\boxed{Y=3}$$

$y=3$

$$\beta_{k+1}(1) = \frac{\overset{\text{sit}\to\text{sit}}{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.6)} + \overset{\text{stand}\to\text{sit}}{\beta_k(2)\left(\frac{\varepsilon}{3}\right)(.2)} + \overset{\text{walk}\to\text{sit}}{\beta_k(3)(1-\varepsilon)(.4)} + \overset{\text{run}\to\text{sit}}{\beta_k(4)\cancel{\left(\frac{\varepsilon}{3}\right)}(0)}}{\beta_k(1)\left(\frac{\varepsilon}{3}\right) + \beta_k(2)\left(\frac{\varepsilon}{3}\right) + \beta_k(3)(1-\varepsilon) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=3$

$$\beta_{k+1}(2) = \frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.1) + \beta_k(2)\left(\frac{\varepsilon}{3}\right)(.4) + \beta_k(3)(1-\varepsilon)(0) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)(.1)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right) + \beta_k(2)\left(\frac{\varepsilon}{3}\right) + \beta_k(3)(1-\varepsilon) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=3$

$$\beta_{k+1}(3) = \frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(.3) + \beta_k(2)\left(\frac{\varepsilon}{3}\right)(.3) + \beta_k(3)(1-\varepsilon)(.3) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)(.6)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right) + \beta_k(2)\left(\frac{\varepsilon}{3}\right) + \beta_k(3)(1-\varepsilon) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$y=3$

$$\beta_{k+1}(4) = \frac{\beta_k(1)\left(\frac{\varepsilon}{3}\right)(0) + \beta_k(2)\left(\frac{\varepsilon}{3}\right)(.1) + \beta_k(3)(1-\varepsilon)(.3) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)(.3)}{\beta_k(1)\left(\frac{\varepsilon}{3}\right) + \beta_k(2)\left(\frac{\varepsilon}{3}\right) + \beta_k(3)(1-\varepsilon) + \beta_k(4)\left(\frac{\varepsilon}{3}\right)}$$

$$\boxed{Y=4}$$

$y=4$

sit→sit stand→sit walk→sit run→sit

$$\beta_{k+1}(1)=\frac{\beta_k(1)(\frac{\varepsilon}{3})(.6)+\beta_k(2)(\frac{\varepsilon}{3})(.2)+\beta_k(3)(\frac{\varepsilon}{3})(.4)+\beta_k(4)\cancel{(1-\varepsilon)}(0)}{\beta_k(1)(\frac{\varepsilon}{3})+\beta_k(2)(\frac{\varepsilon}{3})+\beta_k(3)(\frac{\varepsilon}{3})+\beta_k(4)(1-\varepsilon)}$$

$y=4$

$$\beta_{k+1}(2)=\frac{\beta_k(1)(\frac{\varepsilon}{3})(.1)+\beta_k(2)(\frac{\varepsilon}{3})(.4)+\beta_k(3)(\frac{\varepsilon}{3})(0)+\beta_k(4)(1-\varepsilon)(.1)}{\beta_k(1)(\frac{\varepsilon}{3})+\beta_k(2)(\frac{\varepsilon}{3})+\beta_k(3)(\frac{\varepsilon}{3})+\beta_k(4)(1-\varepsilon)}$$

$y=4$

$$\beta_{k+1}(3)=\frac{\beta_k(1)(\frac{\varepsilon}{3})(.3)+\beta_k(2)(\frac{\varepsilon}{3})(.3)+\beta_k(3)(\frac{\varepsilon}{3})(.3)+\beta_k(4)(1-\varepsilon)(.6)}{\beta_k(1)(\frac{\varepsilon}{3})+\beta_k(2)(\frac{\varepsilon}{3})+\beta_k(3)(\frac{\varepsilon}{3})+\beta_k(4)(1-\varepsilon)}$$

$y=4$

$$\beta_{k+1}(4)=\frac{\beta_k(1)(\frac{\varepsilon}{3})(0)+\beta_k(2)(\frac{\varepsilon}{3})(.1)+\beta_k(3)(\frac{\varepsilon}{3})(.3)+\beta_k(4)(1-\varepsilon)(.3)}{\beta_k(1)(\frac{\varepsilon}{3})+\beta_k(2)(\frac{\varepsilon}{3})+\beta_k(3)(\frac{\varepsilon}{3})+\beta_k(4)(1-\varepsilon)}$$

System B: Accelerometer action (on/off)

$S: \{1, 2, 3, 4\}$
(sit, stand, walk, run)

$U: \{0, 1, 2, 3\}$

1: turn off filter
2: turn on filter 1
3: turn on filter 2

0: turn accelerometer on and get feedback signal

$Y: \{0, 1, 2, 3, 4\}$

~ from accelerometer data
0: action $\neq 0$

Transition Probabilities

~ same as A (independent of action)

Reward Model

- using a filter costs energy ($-e_{filter}$)
- picking the correct action yields a reward R (+)
- Accelerometer ($-e_{accel}$)

$r(s,u)$

$r(s,0) = -e_{accel} \quad \forall s$

$$r(s,1) = \begin{cases} 0 - \cancel{e_{accel}} & s = 3,4 \\ R - \cancel{e_{accel}} & s = 1,2 \end{cases} \quad \text{(No Filter)}$$

$$r(s,2) = \begin{cases} R - e_{filt} - \cancel{e_{accel}} & s = 3 \\ -e_{filt} - \cancel{e_{accel}} & s = 1,2,4 \end{cases} \quad \text{(Filter 1)}$$

$$r(s,3) = \begin{cases} R - e_{filt} - \cancel{e_{accel}} & s = 4 \\ -e_{filt} - \cancel{e_{accel}} & s = 1,2,3 \end{cases} \quad \text{(Filter 2)}$$

accelerometer not on

Observation Model

$$\mathbb{P}(s_{k+1}=j, Y_k=y \mid s_k=i, U_k=u) = \mathbb{P}(Y_k=y \mid s_{k+1}=j, s_k=i, U_k=u)\,\mathbb{P}(s_{k+1}=j \mid s_k=i)$$

$\Rightarrow$ NOT indp. of action, just next state

$$\mathbb{P}(s_{k+1}=j, Y_k=y \mid s_k=i, U_k=u) = \mathbb{P}(Y_k=y \mid s_k=i, U_k=u)\,\mathbb{P}(s_{k+1}=j \mid s_k=i)$$

Define $\mathbb{P}(Y_k=y \mid s_k=i, U_k=u)$

$\underline{u=1,2,3}$

$$\mathbb{P}(Y_k=y \mid s_k=i, U_k=u) = \begin{cases} 1, & y=0 \\ 0, & y\neq 0 \end{cases}$$

$\underline{u=0}$

$$\mathbb{P}(Y_k=y \mid s_k=i, U_k=0) = \begin{cases} 0, & y=0 \\ 1-\varepsilon, & y=i \\ \varepsilon/3, & y\neq i \\ \varepsilon/3, & y\neq i \\ \varepsilon/3, & y\neq i \end{cases}$$

Belief update

$$\beta_{k+1} = \mathbb{B}(\beta_k, U_k, Y_k)$$

$$\beta_{k+1} = \begin{bmatrix} \beta_{k+1}(1) \\ \beta_{k+1}(2) \\ \beta_{k+1}(3) \\ \beta_{k+1}(4) \end{bmatrix} \qquad \beta_{k+1}(j) = \mathbb{P}(s_{k+1} = j \mid s_k \sim \beta_k, U_k = u, Y_k = y)$$

Belief update eqn

$$\beta_{k+1}(j) = \frac{\sum_i \beta_k(i)\, \mathbb{P}(Y_k = y \mid \cancel{s_{k+1} = j}, s_k = i, U_k = u)\, \mathbb{P}(s_{k+1} = j \mid s_k = i, \cancel{U_k = u})}{\sum_i \beta_k(i)\, \mathbb{P}(Y_k = y \mid s_k = i, U_k = u)}$$

$\boxed{U = 1,2,3}$

for all, w.p. $= 1$ $Y_k = 0$

$$\beta_{k+1}(j) = \frac{\sum_i \beta_k(i)\, \underbrace{\cancel{\mathbb{P}(Y_k = y \mid s_{k+1} = j, s_k = i, U_k = u)}}_{1}\, \mathbb{P}(s_{k+1} = j \mid s_k = i, \cancel{U_k = u})}{\underbrace{\cancel{\sum_i \beta_k(i)}}_{\sum \beta(i) = 1}\, \underbrace{\cancel{\mathbb{P}(Y_k = y \mid s_k = i, U_k = u)}}_{1}}$$

$$\beta_{k+1}(j) = \sum_i \beta_k(i)\, \mathbb{P}(s_{k+1} = j \mid s_k = i)$$

$$\beta_{k+1}(j) = \beta_x(1) P_{j|1} + \beta_x(2) P_{j|2} + \beta_x(3) P_{j|3} + \beta_x(4) P_{j|4}$$

~Dependant on j

$j=1$

$$\beta_{k+1}(1) = \overset{\text{sit}\to\text{sit}}{\beta_k(1)(.6)} + \overset{\text{stand}\to\text{sit}}{\beta_k(2)(.2)} + \overset{\text{walk}\to\text{sit}}{\beta_k(3)(.4)} + \overset{\text{run}\to\text{sit}}{\beta_k(4)(0)}$$

$j=2$

$$\beta_{k+1}(1) = \overset{\text{sit}\to\text{stand}}{\beta_k(1)(.1)} + \overset{\text{stand}\to\text{stand}}{\beta_k(2)(.4)} + \overset{\text{walk}\to\text{stand}}{\beta_k(3)(0)} + \overset{\text{run}\to\text{stand}}{\beta_k(4)(.1)}$$

$j=3$

$$\beta_{k+1}(1) = \overset{\text{sit}\to\text{walk}}{\beta_k(1)(.3)} + \overset{\text{stand}\to\text{walk}}{\beta_k(2)(.3)} + \overset{\text{walk}\to\text{walk}}{\beta_k(3)(.3)} + \overset{\text{run}\to\text{walk}}{\beta_k(4)(.6)}$$

$j=4$

$$\beta_{k+1}(1) = \overset{\text{sit}\to\text{run}}{\beta_k(1)(0)} + \overset{\text{stand}\to\text{run}}{\beta_k(2)(.1)} + \overset{\text{walk}\to\text{run}}{\beta_k(3)(.3)} + \overset{\text{run}\to\text{run}}{\beta_k(4)(.3)}$$

$Y=0$ for all, $k=1,2,3$

$\boxed{u=0}$

$$\beta_{k+1}(j) = \frac{\sum_i \beta_k(i)\,\mathbb{P}(Y_k=y \mid \cancel{S_{k+1}=j}, S_k=i, U_k=u)\,\mathbb{P}(S_{k+1}=j \mid S_k=i, \cancel{U_k=u})}{\sum_i \beta_k(i)\,\mathbb{P}(Y_k=y \mid S_k=i, U_k=u)}$$

$\Rightarrow$ same as system A belief update.

Algorithm ~ get $\tilde{A}_0$

$$\tilde{V}_{N-k}(\beta^{(\ell)})$$

$$= \overset{\max}{\cancel{\min_{u\in\mathcal{U}}}} \sum_{i=1}^{4} \beta^{(\ell)}(i)\Big[c_k(i,u) + \sum_{y=1\,0}^{4} \mathbb{P}(Y_k=y \mid S_k=i, U_k=u)\tilde{V}_{N-k-1}(B(\beta^{(\ell)}, u, y))\Big]$$

$$\tilde{V}_{N-k-1}(B(\beta^{(\ell)},u,y)) = \overset{\max}{\cancel{\min_{\alpha\in\tilde{A}_{k+1}}}} \langle B(\beta^{(\ell)},u,y),\alpha\rangle$$

$$\text{new } d_k^{(\ell)} = \Big[c_k(i,u^*) + \underbrace{\sum_{y,j} \overbrace{\mathbb{P}(Y_k=y, S_{k+1}=j \mid S_k=i, U_k=u^*)}^{i\text{ loop}} \cdot d_{k+1}^{(y)}}_{y,j\text{ loop}}\Big]_{i\in S}$$

$y=1:4$
$j=1:4$

where

$$d_{k+1}^{y} = \arg\min_{\alpha\in\tilde{A}_{k+1}} \langle B(\beta^{\ell}, u^*, y), \alpha\rangle \quad \forall y \in \mathcal{Y}$$

gets $A_{k+1}(:, d_{k+1}^{y})$

$\Rightarrow$ $a^0\ a^1\ a^2$ $[\vdots\ \vdots\ \vdots]$ for $y = 0,1,2$

Fill out $A_k(:,\ell)$ with d_k^{ℓ} ~ loop $\ell \to Q$

~ next $k \to 0$, gets $\tilde{A}_0$

$$\text{new } d_k^{(\ell)} = \Big[\overbrace{c_k(i,u^*) + \sum_{y,j} \underbrace{\mathbb{P}(Y_k=y, s_{k+1}=j \mid s_k=i, U_k=u^*) \cdot d_{k+1}^{(y)}}_{y,j \text{ loop}}}^{i \text{ loop}}\Big]_{i \in S}$$

$y = 1:4$
$j = 1:4$

where

$$d_{k+1}^{y} = \underset{d \in \tilde{A}_{k+1}}{\arg\min} \langle B(\beta^{\ell}, u^*, y), d \rangle \quad \forall y \in \mathcal{Y}$$

gets $A_{k+1}(:, d_{k+1}^{y})$

$\Rightarrow$ $d^0 \; d^1 \; d^2$ $\quad [\vdots \; \vdots \; \vdots]$ for $y = 0,1,2$

Fill out $A_k(:,\ell)$ with d_k^{ℓ}

~loop $\ell \to Q$

~ next $k \to 0$

- random vs discrete IB

- size of Q
- size of k

$\left.\begin{array}{l}\text{size of } Q \\ \text{size of } k\end{array}\right\}$ find convergence

should avg out quick

k ≈ 100 should be ok

change Q small → large

until converge

increase k?

good or bad.

~ maybe try 2 to 3 states first.